GAZETAS, G. (1981). *Géotechnique* **31**, No. 3, 387–398

Machine foundations on deposits of soft clay overlain by a weathered crust

G. GAZETAS*

Currently available methods to design machine foundations against excessive dynamic displacements assume that the stiffness of the supporting soil is either constant or increases with depth. But frequently, soft normally consolidated clay deposits have stiffer upper crusts due to desiccation and weathering; in such crusts shear modulus and strength decrease sharply with depth. The Paper presents a theoretical study of the coupled horizontal-rotational and vertical vibrations of long massive foundations that carry rotating machinery and are supported by such weathered clay deposits. A key step in the analysis is the determination of the dynamic foundation impedances appropriate for the range of anticipated operational frequencies of the machinery. By utilizing a rational analytical procedure, extensive parametric studies were carried out and are reported in the form of graphs relating normalized impedance functions to crucial non-dimensional factors influencing the response. Particular emphasis is accorded to the effects of the degree and depth of weathering and a numerical example illustrates the direct practical applicability of the graphs.

Les méthodes dont on dispose actuellement pour calculer des fondations de machines sans déplacements dynamiques excessifs admettent que la rigidité du sol support est constante ou qu'elle augmente avec la profondeur. Mais bien souvent, less croûtes supérieures des dépôts d'argile molle normalement consolidée sont plus rigides en raison de la dessiccation et de l'altération; dans le cas de telles croûtes, le module de cisaillement et la résistance à la rupture au cisaillement diminuent brusquement en fonction de la profondeur. L'article présente une étude théorique des vibrations verticales et horizontales-circulaires associées auxquelles sont soumises de longues fondations massives qui portent des machines rotatives et reposent sur de tels dépôts d'argile 'altéré'. L'étape-cléf de l'analyse réside dans la détermination des 'impédances' dynamiques de fondations convenant à la gamme de fréquences opérationnelles prévues pour les machines. A l'aide d'une méthode analytique rationnelle, d'importantes études paramétriques ont été effectuées; les résultats de ces études sont présentés sous forme de graphiques qui représentent les fonctions d'impédances normalisées par rapport aux facteurs critiques adimensionnels qui influencent la réponse. Une importance particulière est accordée aux effets du degré et de la profondeur de l'alteration, et l'application pratique directe des graphiques présentés est illustrée à l'aide d'un exemple numérique.

Discussion on this Paper closes 1 December, 1981. For further details see inside back cover.

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NOTATION

A_s	frequency factor $= 2fB\sqrt{(G_s/\rho_s)}$
B	foundation halfwidth
C_v, C_u, C_r	normalized dynamic impedances (equations (8)–(11))
d	eccentricity of unbalanced mass
D_{cr}	depth of crust (Fig. 2)
f	operational frequency in c/s
G_{cr}	maximum shear modulus of crust (Fig. 2)
G_s	shear modulus of soft clay layer
H	total thickness of soil deposit to rock
I	total moment of inertia of foundation and machine about centre of gravity
K	ratio of shear modulus over undrained shear strength of clays
m	total mass of foundation and machine
m_0	unbalanced mass of machine
M_r	exciting force (acting at foundation base) resulting from dynamic impedances
Q_v, Q_u	exciting forces (acting at centre of gravity) resulting from dynamic impedances
r	rotation of a foundation
R_v, R_u	soil reactions (acting at foundation base), resulting from dynamic impedances
s_u	undrained shear strength of soil
t	time
T_r	soil reaction (acting at foundation base) resulting from dynamic impedances
u	horizontal displacement of a foundation
v	vertical displacement of a foundation
ρ	mass density
ω	operational circular frequency $= 2\pi f$

INTRODUCTION

In practice one very often encounters soil profiles consisting mainly of normally or slightly overconsolidated soft clays whose upper part has been exposed to desiccation and weathering, probably as a consequence of a fall in sea or lake water level. Desiccation not only causes development of pore-water tensions which increase the effective stresses in the soil, but it also encourages chemical weathering, especially oxidation, which gives the soil an apparent overconsolidation. Consequently, the strength and stiffness of the upper soil increase, thereby forming a (fissured) crust that is underlain by softer clay. The stiffness and thickness of such a crust depend on many factors: climate, soil perme-

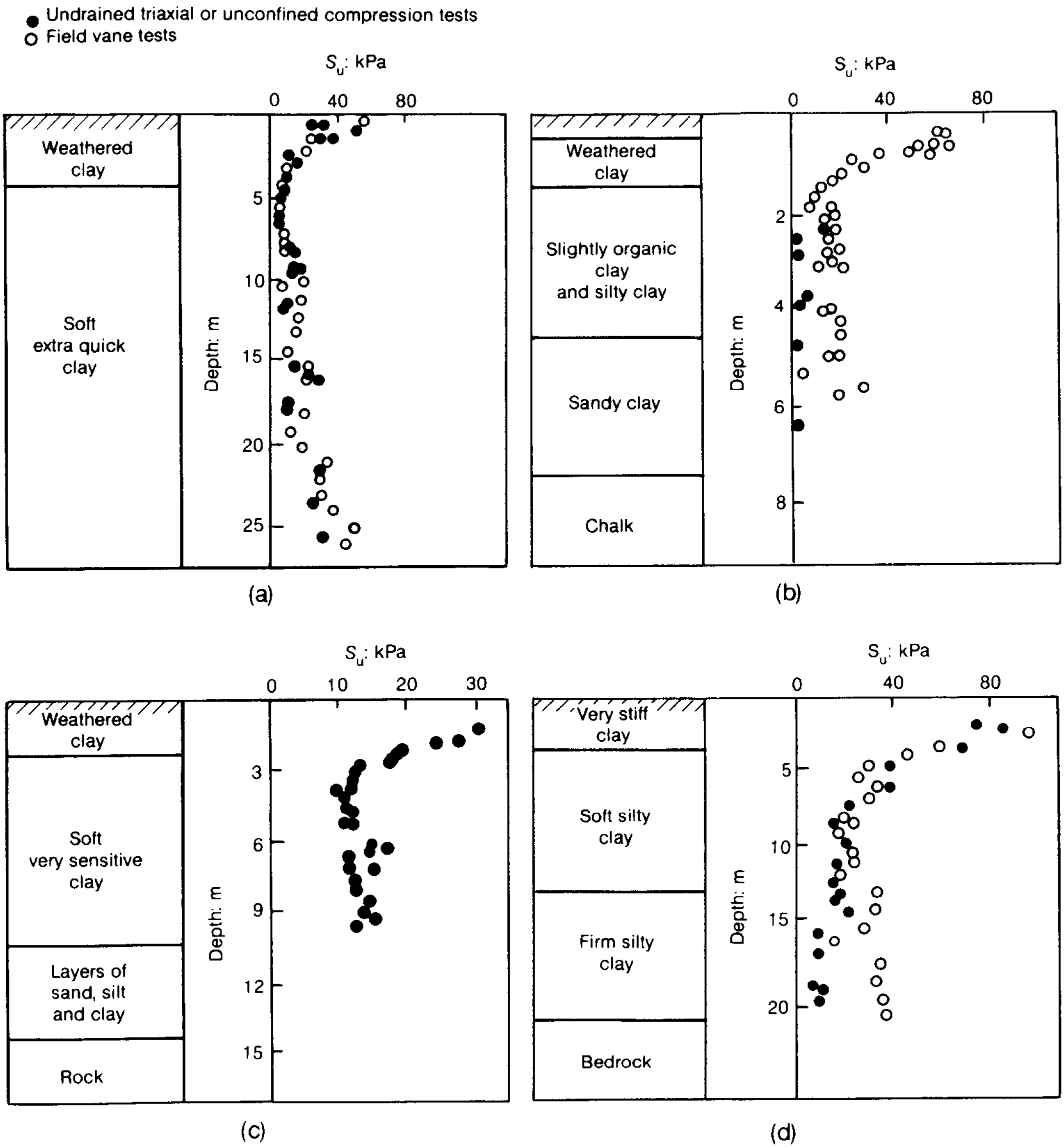


Fig. 1. Typical deposits of soft clay with a weathered upper crust; from (a) Oslo, (b) Yorkshire, (c) New Hampshire and (d) Ontario

ability, chemical synthesis of the clay, vegetation and so on.

Four typical soil profiles exhibiting weathered upper crust are portrayed in Fig. 1. They were recorded in four countries: Norway (Bjerrum, 1954, pp. 56, 59), England (Symons & Murray, 1971), USA (Simon, Christian & Ladd, 1974) and Canada (Lo & Becker, 1979). Their common characteristic is the rapid decrease with depth of the undrained shear strength s_u in the crust. In contrast, the unweathered clay exhibits a constant or slightly increasing s_u with depth.

The variation with depth of the soil stiffness, expressed in terms of the shear modulus G, or the undrained Young's modulus E_u, is expected to be similar. This is because these moduli are related to the undrained shear strength (approximately) in the form

$$G \simeq K s_u \tag{1}$$

where the constant of proportionality K is chiefly a function of the shearing strain amplitude. For

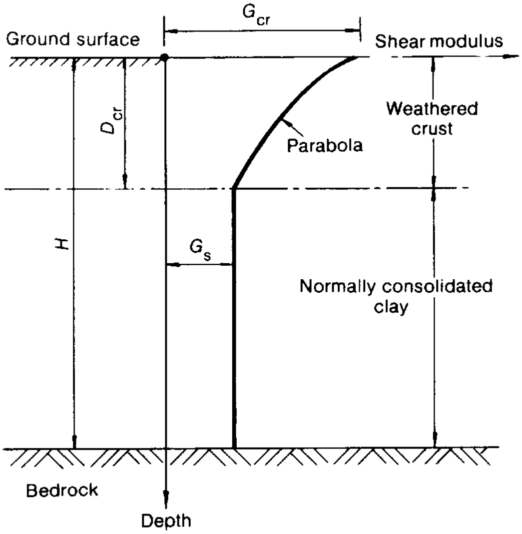


Fig. 2. Idealized soil profile used in this study

instance, Seed & Idriss (1970) reported an average value of $K = 2300$ for very small strains (less than 10^{-6}) while Ladd (1964), Hara *et al.* (1974) and Simon *et al.* (1974) suggested values of about 500 for larger amplitudes of strain. Recently, the Applied Technology Council (1978) recommended $K \simeq 1000$.

Consequently a soil profile with a distribution of modulus with depth such as the one depicted in Fig. 2 may be a very realistic idealization for many actually encountered soil deposits. Yet no published study exists on the dynamic behaviour of foundations resting on such soils. Thus the practising geotechnical engineer involved in the design of machine foundations has little choice other than to use available solutions developed for homogeneous soils (e.g. Kobori, Minai & Suzuki, 1971; Beredugo & Novak, 1972; Luco, 1974; Kausel & Roesset, 1975; Gazetas & Roesset, 1976). To estimate an effective modulus to be used in such analyses, empirical rules of thumb have been proposed. For example, Whitman (1976) has recommended the use of the average modulus over a depth equal to the foundation radius (or halfwidth) for rocking, torsion and horizontal translation, and over one and a half times the radius (or halfwidth) for vertical translation. He emphasized, however, that considerable judgement must be exercised in the application of such empirical rules. In fact, the Author (using the method outlined by Gazetas, 1980) has shown that such rules lead to an underprediction by a factor of at least 2 of the rocking vibration amplitudes experienced by a strip foundation resting on a deposit whose shear wave velocity increases linearly with depth. With more complicated forms of modulus variation with depth, as that of Fig. 2, the need for a rational evaluation of the dynamic response of foundations is even more needed.

In response to this need, this Paper offers the results of a series of parametric analytical studies for a rigid strip founded on a deposit of soft clay overlain by a stiff crust (Fig. 2). The results are cast in the form of graphs relating normalized load–displacement ratios (i.e. stiffnesses) to key dimensionless factors that influence the foundation response to dynamic normal, shear and moment loading. It is believed that this study fills a significant gap in the soil dynamics literature.

DYNAMICS OF MASSIVE MACHINE FOUNDATIONS

Figure 3(a) shows in perspective a long massive foundation supporting a number of rotating machinery in series. This arrangement is typical for a variety of machines (such as centrifugal pumps and turbo-generators) and is the only feasible one for some special types of machinery (such as wire standers[1]). The response of such long foundations to the harmonic forces and moments generated by unbalanced rotating masses is investigated here. The soil profile is of the form shown in Fig. 2.

With the foundation length being much larger (by a factor of about 4 or more) than the foundation width, essentially plane-strain conditions prevail in the soil, except near the ends of the footing. In such cases a two-dimensional approximation of the actual geometry has been found to lead to reasonably good results[2] (see Valera *et al.*, 1977). With the notation given in Fig. 3(b), which portrays a cross-section of the foundation, the equations of motion in vertical translation $v(t)$, horizontal translation $u(t)$ and rotation $r(t)$, referred to the centre of gravity of the foundation–machine system, are

$$m\ddot{v}(t) + R_v(t) = Q_v(t) \tag{2}$$

$$m\ddot{u}(t) + R_u(t) = Q_u(t) \tag{3}$$

$$I\ddot{r}(t) + T_r(t) - R_u(t)\,z_c = M_r(t) \tag{4}$$

in which m = total foundation mass (per unit length); I = mass moment of inertia (per unit length) about the horizontal axis passing through the centre of gravity; $R_v(t)$, $R_u(t)$ and $T_r(t)$ = vertical, horizontal and moment reactions of the soil acting at the foundation base (per unit length); $Q_v(t)$, $Q_u(t)$ and $M_r(t)$ = vertical, horizontal and moment excitations (per unit length), respectively, acting at the

[1] Machines forming wires or ropes by twisting together strands.

[2] In fact, widely used dynamic finite element computer codes are based on a two-dimensional discretization of the soil–foundation system (see Valera *et al.*, 1977 for a list of references on the subject).

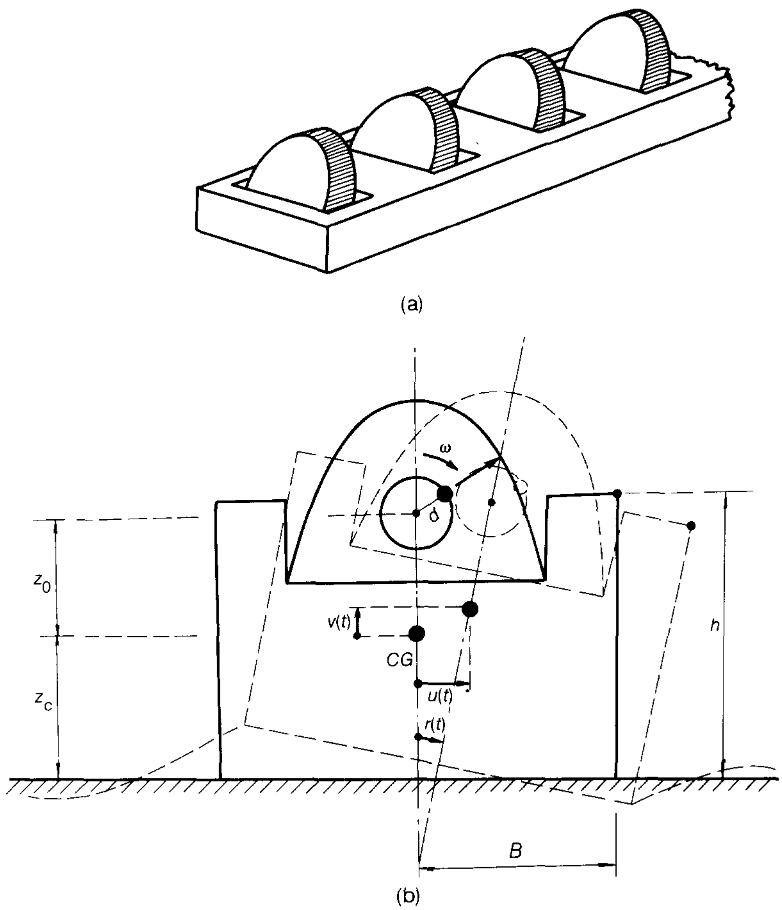


Fig. 3. Perspective and cross-section of rotating machine foundation; definition of geometric and deformational parameters

centre of gravity. The dots denote differentiation with respect to time t.

If the excitation arises from an unbalanced mass m_0 (per unit length) rotating with a (small) eccentricity d at the operational frequency $\omega = 2\pi f$, the excitation forces can be written in complex notation as follows

$$Q_v = m_0 d\omega^2 \exp[i(\omega t + 90°)] \quad (5)$$

$$Q_u = m_0 d\omega^2 \exp(i\omega t) \quad (6)$$

$$M_r = z_0 m_0 d\omega^2 \exp(i\omega t) \quad (7)$$

in which $i = \sqrt{-1}$ and z_0 = distance of the rotating mass from the centre of gravity (Fig. 3(b)).

The soil reactions R_v, R_u and T_r are related to the vertical, horizontal and rotational displacements of the soil–foundation interface that result from the vibratory motion of the ground. For the small amplitudes of strain (usually of the order of 10^{-5} or less) imposed by machine foundations, soil behaves as a linear elastic medium and the relationships between soil reactions and displacements may be expressed in the form

$$R_v(t) = G_s(C_{v1} + iC_{v2})\, v(t) \quad (8)$$

$$R_u(t) = G_s(C_{u1} + iC_{u2})\,[u(t) - z_c r(t)] \quad (9)$$

$$T_r(t) = G_s B^2(C_{r1} + iC_{r2})\, r(t) \quad (10)$$

in which G_s = shear modulus of the unweathered clay layer (Fig. 2) and B = half the width of the foundation. The stiffness terms (also called impedances)

$$C_a = C_{a1} + iC_{a2}, \quad a = v, u, r \quad (11)$$

are complex numbers whose real and imaginary parts are both functions of frequency ω. The real part reflects the stiffness and inertia of the soil; its dependence on frequency is attributed solely to the influence which frequency exerts on inertia,[3] since soil properties (and hence soil stiffness) are independent of frequency[4] (Richart, Woods & Hall, 1970). The imaginary part reflects the radiation and internal damping of the medium. The former is the result of energy dissipation by waves propagating away from the foundation and never rebounding, and is frequency dependent; whereas the latter stems mainly from the hysteretic behaviour of soil under cyclic loading and is practically independent of frequency.

A rigorous analytical–numerical procedure has been utilized in order to determine, for each operational frequency, the vertical, horizontal and rotational impedances for a rigid strip resting on the surface of an idealized soft clay deposit with a weathered crust (Fig. 2). Besides the assumption of linear viscoelastic soil behaviour no other simplifying assumptions are made. The soil is divided into its two natural layers: the crust, whose modulus varies with depth according to

$$G = G_{cr}(1-bz)^2, \quad 0 \leqslant z \leqslant D_{cr} \tag{12}$$

in which $b = [1-\sqrt{(G_s/G_{cr})}]/D_{cr}$; and the soft clay, with

$$G = G_s, \quad D_{cr} < z < H \tag{13}$$

Poisson's ratio ν and mass density ρ are constant within each layer. No limitation is placed on the total thickness of the deposit H, which may take values between a maximum value of $H = \infty$ (half-space) and a minimum of $H = D_{cr}$ (stratum of decreasing stiffness on bedrock).

The theoretical formulation and solution of this elastodynamic boundary value problem has been presented in detail elsewhere (see Gazetas, 1980), for an arbitrary number of soil layers. The method, based on a transformation that leads to a closed-form solution of the coupled wave equations for each inhomogeneous layer, is exact in that it properly accounts for the true boundary conditions at all layer interfaces and the ground surface.

Once the three impedances, C_v, C_u and C_r, have been obtained, substitution of equations (8)–(10) in equations (2)–(4) leads to three equations where the only unknowns are the displacements $v(t)$ and $u(t)$ and the rotation $r(t)$. It appears that horizontal and rocking vibrations are coupled (equations (3), (4)) and independent of the vertical vibration (equation (2)). With the excitation forces described by equations (5)–(7), the steady state motions are

$$v(t) = v_c \exp[i(\omega t + 90°)] \tag{14}$$

$$u(t) = u_c \exp(i\omega t) \tag{15}$$

$$r(t) = r_c \exp(i\omega t) \tag{16}$$

in which v_c, u_c and r_c are complex, frequency-dependent displacement and rotation amplitudes given, respectively, by

$$v_c = \frac{m_0\, d\omega^2}{G_s C_v - m\omega^2} \tag{17}$$

$$u_c = \frac{m_0\, d\omega^2}{N(\omega)}\{G_s[B^2 C_r + z_c(z_0+z_c)C_u] - I\omega^2\} \tag{18}$$

$$r_c = \frac{m_0\, d\omega^2}{N(\omega)}\{G_s(z_0+z_c)C_u - z_0 m\omega^2\} \tag{19}$$

where

$$N(\omega) = (G_s C_u - m\omega^2)[G_s(B^2 C_r + z_c{}^2 C_u) - I\omega^2] - (G_s z_c C_u)^2 \tag{20}$$

Having determined the real and imaginary parts of each complex displacement from equations (17) to (19), one can compute the amplitude of vibration and the phase angle between dynamic force and displacement. For example, if equation (18) is written as

$$u_c = u_{c1} + iu_{c2} \tag{21}$$

then

$$\text{amplitude of } u_c = \sqrt{(u_{c1}{}^2 + u_{c2}{}^2)} \tag{22}$$

$$\text{phase angle} = \arctan(u_{c2}/u_{c1}) \tag{23}$$

From the motion of the centre of gravity, the horizontal and vertical components of the motion experienced by any point of the foundation or machine can be readily computed. The upper edge of the footing, for instance, experiences

$$v_e = Br_c + v_c \exp(i\pi/2) \tag{24}$$

$$u_e = u_c + (h - z_c)r_c \tag{25}$$

In conclusion, it is evident from the preceding analysis that the successful evaluation of the performance of a machine foundation hinges at the realistic estimation of the dynamic impedances, C_v, C_u and C_r, for the soil profile, footing geometry and range of operational frequencies of the specific situation.

IMPEDANCE FUNCTIONS: SOFT CLAY WITH STIFF CRUST

With the notation of Fig. 2, the dimensionless parameters that influence the dynamic impedances are

(a) the frequency factor $A_s = 2fB\sqrt{(G_s/\rho_s)}$, where f = operational frequency in c/s, G_s and ρ_s = shear modulus and density of the soft clay layer

[3] With harmonic motions, inertia forces are proportional to ω^2.

[4] This is true for a wide range of frequencies, such as those of interest in machine foundation design; it ceases to hold true for the high loading speeds encountered, for example, during blasting operations.

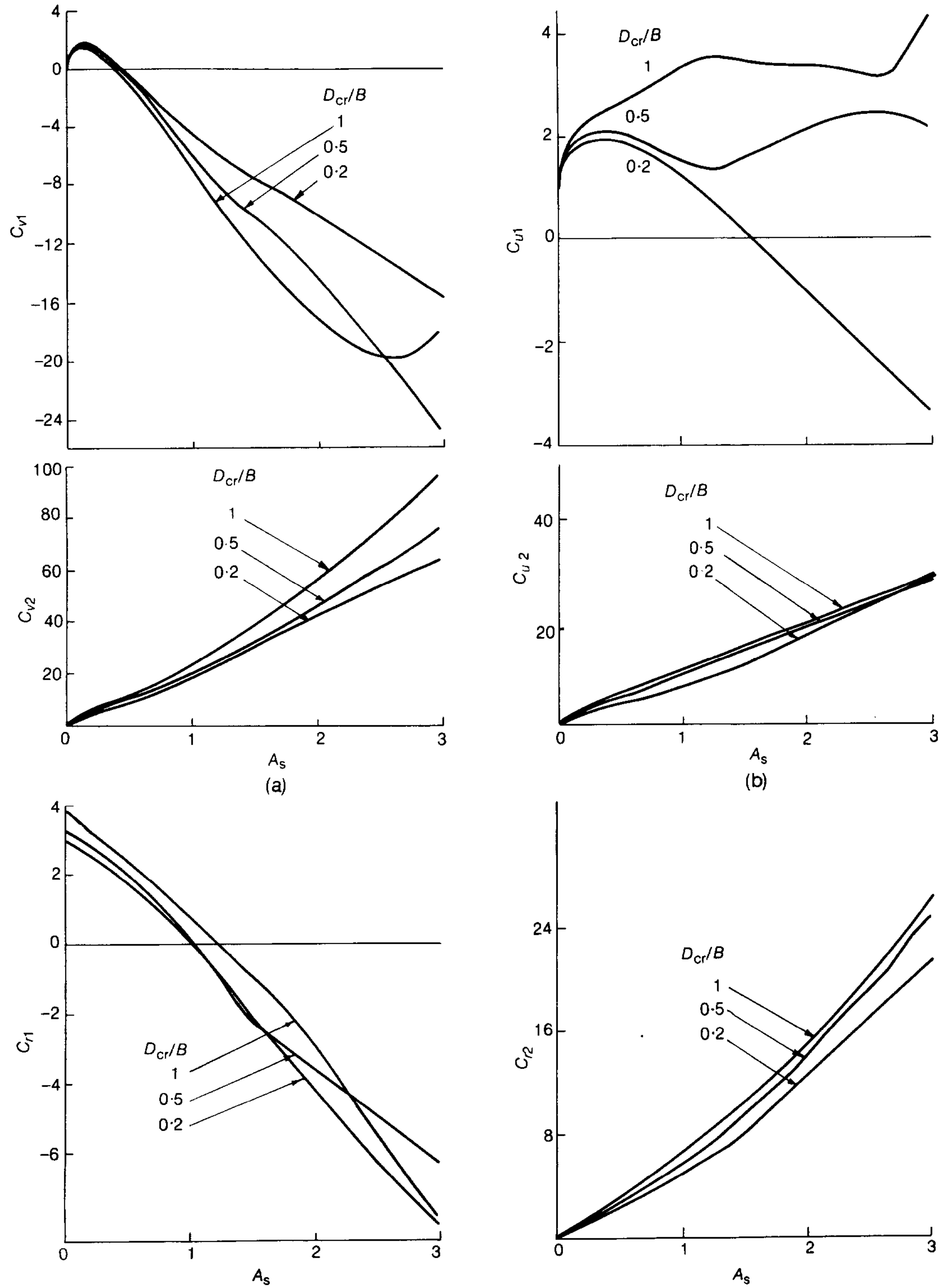


Fig. 4. Dynamic impedance functions for deposits with $G_{cr}/G_s = 2$, $H/B = \infty$ and variable crust thickness; (a) vertical, (b) horizontal and (c) rotational

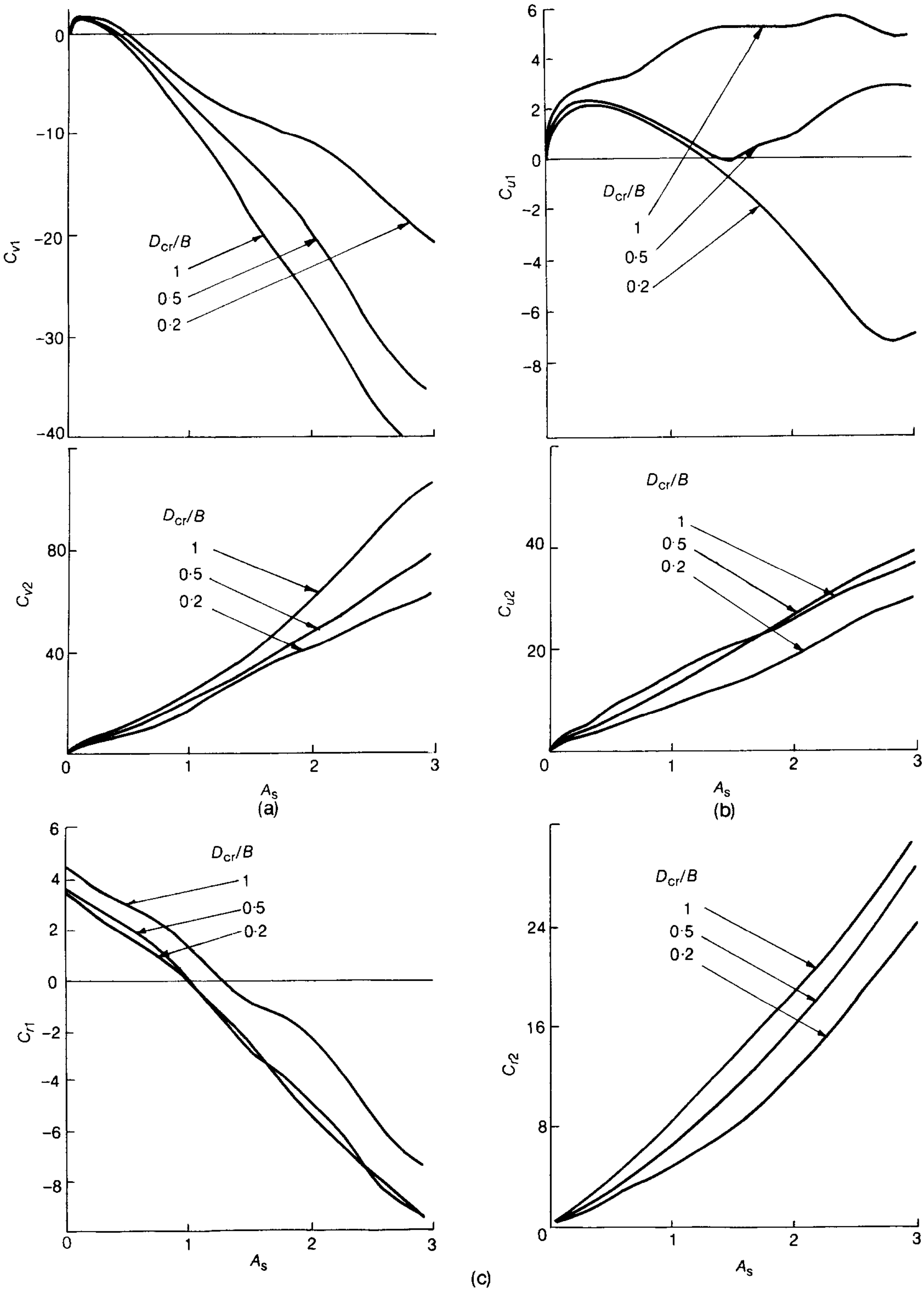
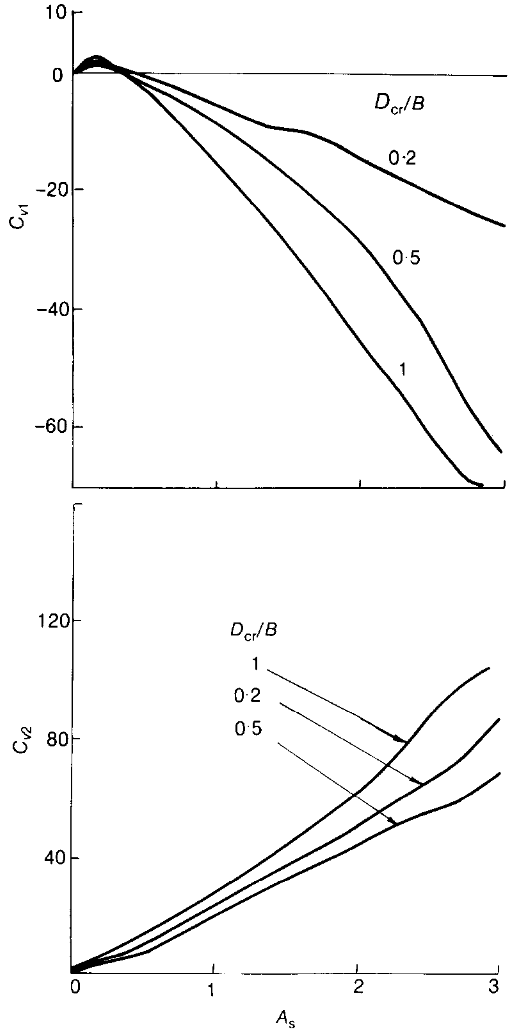
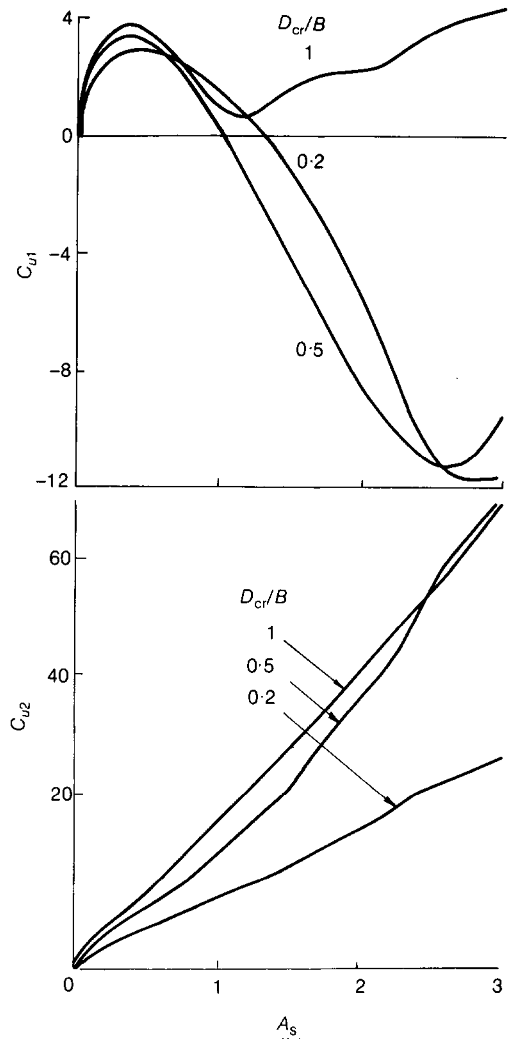


Fig. 5. Dynamic impedance functions for deposits with $G_{cr}/G_s = 4$, $H/B = \infty$ and variable crust thickness; (a) vertical, (b) horizontal and (c) rotational



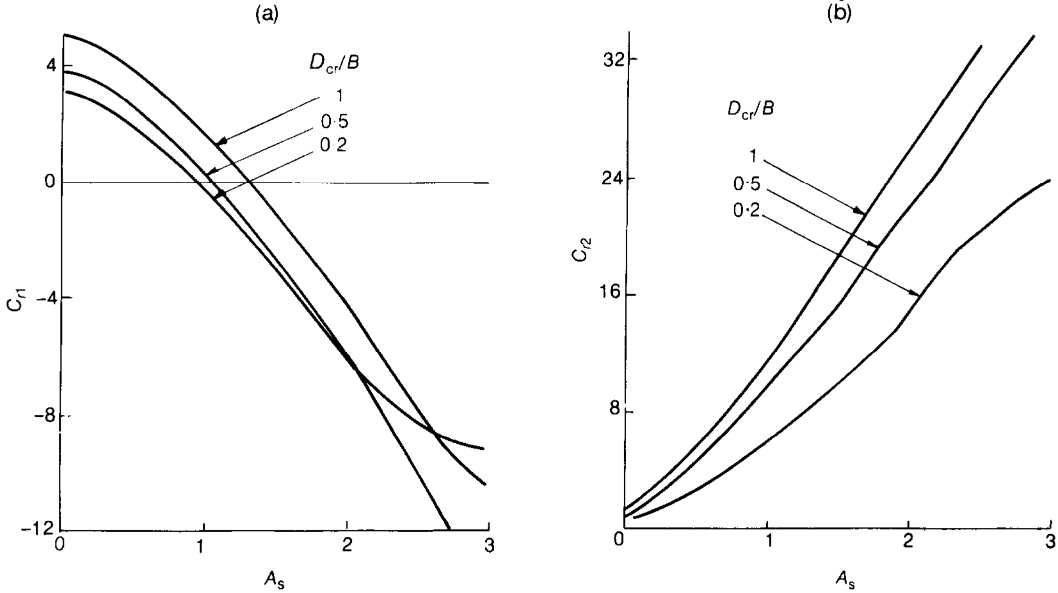
10
0
-20
-40
-60
C_{v1}
D_{cr}/B
0·2
0·5
1
120
80
40
C_{v2}
D_{cr}/B
1
0·2
0·5
0
1
2
3
A_s

(a)



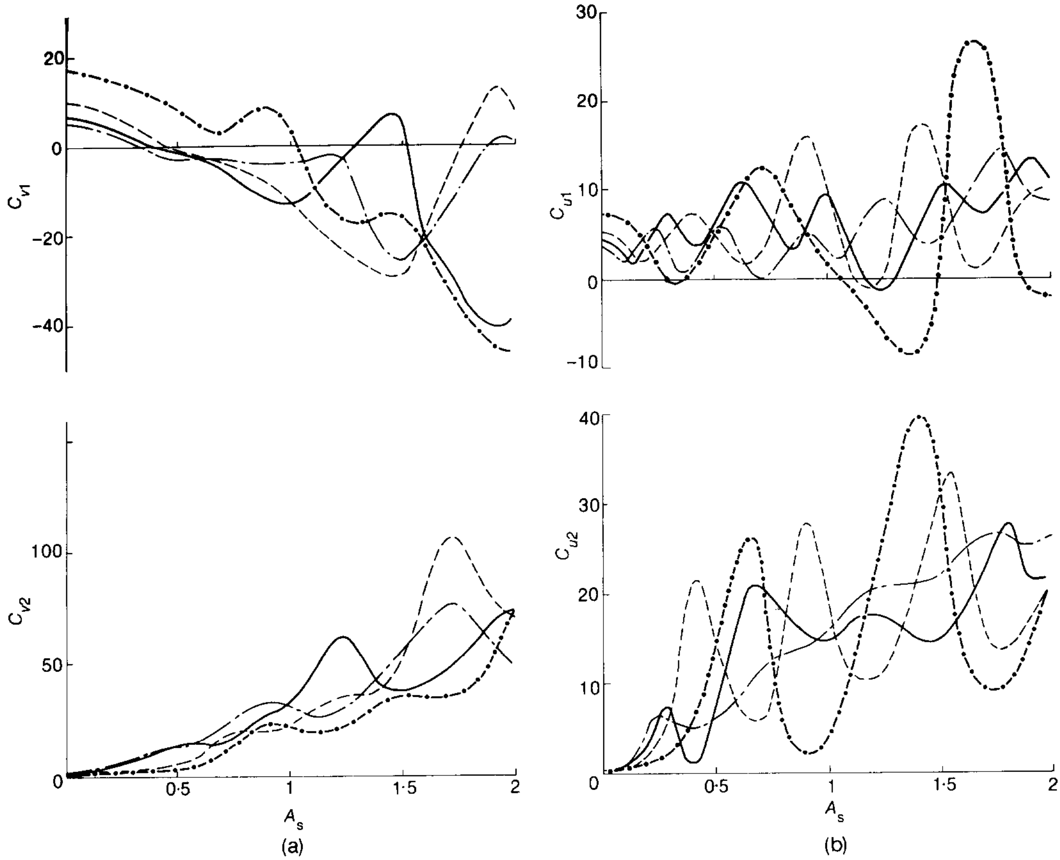
4
0
-4
-8
-12
C_{u1}
D_{cr}/B
1
0·2
0·5
60
40
20
C_{u2}
D_{cr}/B
1
0·5
0·2
0
1
2
3
A_s

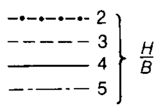
(b)



4
0
-4
-8
-12
C_{r1}
D_{cr}/B
1
0·5
0·2
0
1
2
3
A_s
32
24
16
8
C_{r2}
D_{cr}/B
1
0·5
0·2
0
1
2
3
A_s

(c)





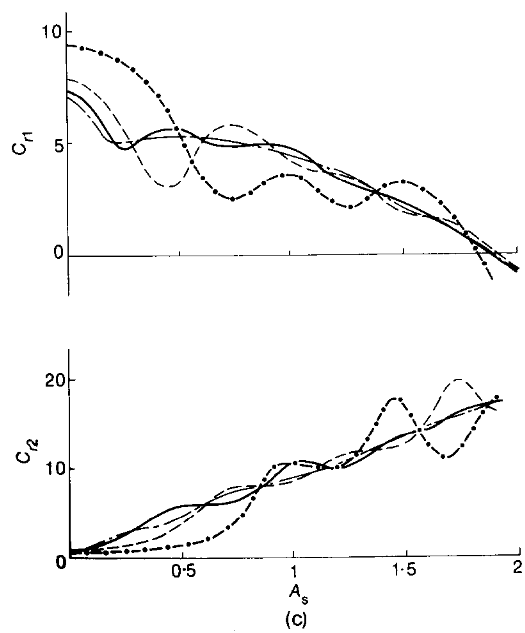


Fig. 7. Dynamic impedance function for deposits with $G_{cr}/G_s = 4$, $D_{cr}/B = 2$ and variable thickness to bedrock; (a) vertical, (b) horizontal and (c) rotational

Fig. 6 (opposite page) Dynamic impedance functions for deposits with $G_{cr}/G_s = 14$, $H/B = \infty$ and variable crust thickness; (a) vertical, (b) horizontal and (c) rotational

(b) the moduli ratio G_{cr}/G_s, which is an index of the degree of weathering; G_{cr} denotes the maximum shear modulus at the surface of the crust (Fig. 2)
(c) the crust-thickness to foundation-halfwidth ratio D_{cr}/B which is an index of the depth of weathering
(d) the depth-to-rock to foundation halfwidth ratio H/B
(e) the hysteretic critical damping ratio for the soil which, in this study, was held constant equal to 3% for both layers
(f) Poisson's ratio ν of the soil; different values of ν were assigned to the stiff crust and the soft clay layer, in consistency with their most probable drainage characteristics: $\nu_{cr} = 0{\cdot}25$ and $\nu_s = 0{\cdot}45$

The results are presented in the form of four sets of figures (Figs 4–7). Each figure consists of three parts portraying the vertical, horizontal and rotational impedances as functions of the frequency factor A_s for several typical values of the D_{cr}/B or H/B parameters. Both real and imaginary parts are shown.

The effect of the degree and extent of weathering is studied in Figs 4–6, where it is assumed that the deposit is very deep ($H/B \to \infty$). Each figure pertains to a single value of the moduli ratio: $G_{cr}/G_s = 2$ in Fig. 4, $G_{cr}/G_s = 4$ in Fig. 5 and $G_{cr}/G_s = 14$ in Fig. 6. Three different curves are plotted in each figure; they correspond to $D_{cr}/B = 0{\cdot}20$, $0{\cdot}50$ and $1{\cdot}00$.

It is evident that weathering has a pronounced effect on all impedances. Especially sensitive to changes in the moduli and depth ratios are the horizontal impedances, whereas the vertical and rocking ones are somewhat less affected.

Furthermore the weathering effects exhibit a strong dependence on frequency. For example, at low frequency factors vertical impedances are relatively indifferent to variations (within realistic limits) in either stiffness or depth of the crust. This is understandable in view of the fact that vertical surface strip-loading affects the soil at great depths, of the order of $8B$ (e.g. Schmertmann, Hartman & Brown, 1978); thus a stiff crust with $D_{cr} \leqslant B$ can only be of secondary importance. This picture, however, changes at higher frequency factors, as may be seen in Figs 4–6. Greater participation of surface (Rayleigh) waves in the motion and stronger reflection of the body waves emanating from the foundation by the soft layer interface, may be explanations of the phenomenon.

Rocking impedances show about the same sensitivity to weathering throughout the frequency range examined and, in general, the imaginary parts of all three impedance functions exhibit only a small dependence on either G_{cr}/G_s or D_{cr}/B.

Finally, the influence of the presence of bedrock at relatively shallow depths is demonstrated in Fig. 7. The soil profile consists of a stiff crust of depth $D_{cr} = 2B$ and shear modulus $G_{cr} = 4G_s$. Four different depths to bedrock are considered: $H/B = 2, 3, 4, 5$. It is evident that as H/B decreases (stratum becomes shallower) the stiffness of the medium increases in the low frequency range, in agreement with static elasticity. At higher frequency factors resonance phenomena occur due to reflection of the downward propagating waves by the rigid bedrock. As a consequence, the impedance functions exhibit fluctuations (peaks and valleys) that are not observed in the case of very deep deposits. As H/B increases, these fluctuations become increasingly smooth (especially with rocking) and the differences of the actual impedance functions from those obtained for an infinitely deep deposit (halfspace) decrease. (See Kausel & Roesset (1975) and Gazetas (1980, 1981) for a more detailed discussion of these phenomena.)

EXAMPLE

The following example demonstrates the practical applicability of the developed graphs in the analysis and design of long massive foundations carrying machinery with rotating unbalanced masses.

A series of large centrifugal pumps are to be founded through a rigid foundation on a soil deposit consisting of about 20 m of normally consolidated clay having a 1 m thick upper weathered crust. Laboratory resonant column tests on undisturbed samples and in situ cross-hole tests give an average value for the shear modulus of the soft clay of $G_s \simeq 40{\cdot}5$ MPa and for the top of the crust of $G_{cr} \simeq 4G_s$. The density of the soft clay is found to be $\rho_s \simeq 1{\cdot}80\ \mathrm{t/m^3}$ and of the crust $\rho_{cr} \simeq 2{\cdot}10\ \mathrm{t/m^3}$.

The machines contain unbalanced masses and may operate at selected frequencies from 2 c/s to 50 c/s. On the basis of data provided by the manufacturer, it is estimated that the unbalanced mass-moment (per unit of foundation length) is $m_0 d = 0{\cdot}0067$ t m/m.

Four alternative foundation schemes have been proposed and their anticipated performance is analysed here. All foundations are long, rigid, 4 m wide (i.e. $B = 2$ m) blocks of concrete, differing primarily with respect to their inertia characteristics, ranging from the light and squat foundation A to the heavy and slender foundation D. Table 1 depicts the geometric and material parameters of the four foundations needed for the analysis.

For each operational frequency, the response of each foundation is evaluated on the basis of equations (17)–(23) (displacements and rotation of the centre of gravity) and equations (24) and (25) (displacements of the upper corner of the founda-

Table 1. Parameters of four alternative trial foundation schemes: $B = 2\,m$

Parameter	Foundation A	B	C	D
m: t/m	20·0	40·0	40·0	60·0
I: $t\,m^2/m$	30·0	90·0	60·0	260·0
z_c: m	1·0	2·0	2·0	3·0
h: m	2·0	4·0	4·0	6·0
z_0: m	2·0	3·0	2·5	4·0

tion). The dynamic impedances to be introduced in these equations are obtained from Fig. 5 pertaining to an infinitely deep ($H/B = \infty$) clay deposit with an upper crust of top modulus $G_{cr} = 4G_s$. The choice of these results is very reasonable since, in this case, $H/B = 20/2 = 10$ and thus the effect of the bedrock on the response would be negligible.

For example, at $f = 10$ c/s the frequency factor is

$$A_s = 2fB\sqrt{(G_s/\rho_s)}$$
$$= 2 \times 10 \times 2 \times \sqrt{(40\,500/1{\cdot}8)} \simeq 0{\cdot}27$$

and one reads from the $D_{cr}/B = 1/2$ curve of Fig. 5: $C_{v1} = 1{\cdot}05$, $C_{v2} = 4{\cdot}61$, $C_{u1} = 2{\cdot}34$, $C_{u2} = 3{\cdot}74$, $C_{r1} = 2{\cdot}81$ and $C_{r2} = 1{\cdot}41$. Then equations (17)–(25) will yield for the centre of gravity of, say, foundation A: $|v_c| = 0{\cdot}129\,m \times 10^{-3}$, $|u_c| = 0{\cdot}388\,m \times 10^{-3}$ and $|r_c| = 0{\cdot}201\,m \times 10^{-3}$ and for its upper corner: $|v_e| = 0{\cdot}507\,m \times 10^{-3}$ and $|u_e| = 0{\cdot}581\,m \times 10^{-3}$. The phase difference of the last two displacements is 27·825°. Similarly one may compute the response at other frequencies, for every foundation.

Figure 8 portrays the variation with frequency of the amplitude of horizontal vibrations at the top corner of each foundation. It appears that increasing the mass and moment of inertia of the foundation–machine system, in this particular case, does not substantially reduce its peak amplitudes. However, a significant decrease of the resonant frequencies does take place and thus the performance improves with increasing inertia (i.e. from A to D). This is because the limiting displacement amplitudes for the safe operation of a machine are not constant but decrease with increasing frequency. A typical safe response spectrum (adapted from Richart *et al.*, 1970) is also plotted in Fig. 8 to allow a direct evaluation of the performance of each scheme. The superiority of foundation D is evident, although at high frequencies its performance is clearly unacceptable.

CONCLUSIONS

The Paper has studied the coupled horizontal-rotational and vertical vibrations of rigid long rectangular foundations that carry rotating machinery and rest on deposits of normally con-

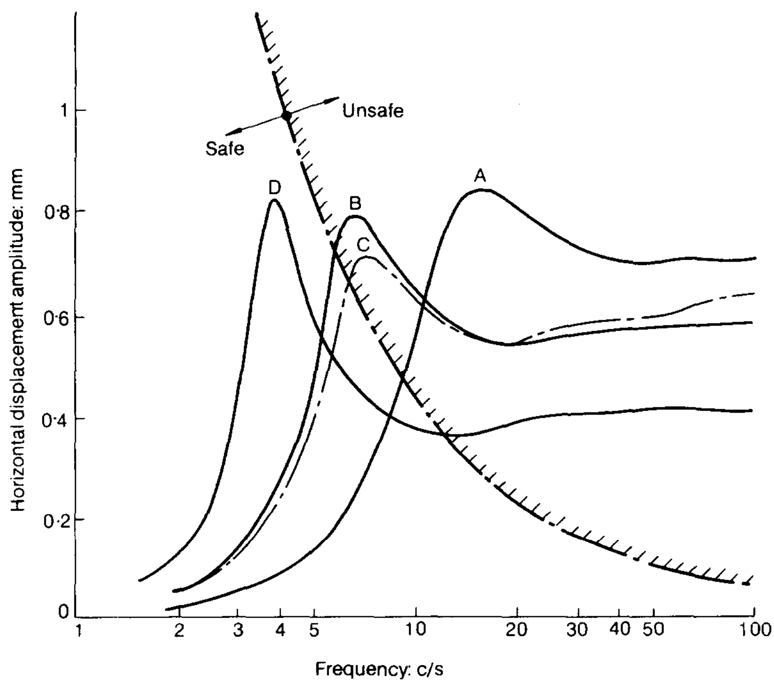


Fig. 8. Predicted response curves of four alternative foundations carrying a series of centrifugal pumps; the upper limit spectrum for safe machine operation is also plotted for comparison

solidated clay whose upper part has been stiffened by desiccation and weathering. A basic step in the analysis is the analytical evaluation of the so-called dynamic impedances of the particular soil–footing system, for the range of anticipated operational frequencies of the machines.

A number of parameteric studies are reported in the form of readily usable graphs portraying three sets of normalized impedances as functions of the dimensionless frequency factor A_s, the ratio of shear moduli at the top and bottom of the crust G_{cr}/G_s, the crust-depth to foundation-halfwidth ratio D_{cr}/B, and the total deposit-thickness to foundation-halfwidth ratio H/B. Realistic values have been assigned to the other independent variables of the problem, namely the Poisson's ratios, critical internal damping ratios and densities of the two soil layers.

The practical applicability of the presented graphs is demonstrated through a numerical example referring to the design of a long foundation that would carry a number of large centrifugal pumps which may operate at a fairly wide range of frequencies. Proposed alternative foundation schemes are easily evaluated by analysing their response to the anticipated level of excitation.

It is believed that the developed parametric data can be adopted to the needs of many practical situations involving soft clay deposits with stiff upper crusts. This fills an apparent gap in the existing soil dynamics literature.

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